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Designing a Local Coordinate System

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A local coordinate system is one that treats a portion of the earth's surface as if it was flat and allows spatial data users to identify the location of a point on the earth with simple plane coordinates. It can be either two-dimensional (2-D) or three-dimensional (3-D) and is essentially the conventional Cartesian coordinate system that has been around since Rene Descartes published his discourse on geometry in 1637.

The most common plane coordinate system used by surveyors is a simple tangent plane touching the earth at a point selected by the user. Other details include assumed coordinates assigned to the origin and some reference direction – usually north. An article entitled, "Coordinates, Calculators, and Intersections" is published in the March 1986 issue of the ACSM *Surveying & Mapping Journal*, Vol. 46, No. 1, pp 29-39. It describes features of the Math/Science Reference System and the Surveyor's Reference System and shows how they are used for routine coordinate geometry (COGO) computations. That article is strictly 2-D COGO as applied to surveying computations using a standard scientific calculator, spreadsheet, or computer.

But, as everyone knows, the earth is not flat and a tangent plane system distorts horizontal angles and distances more and more as one gets further from the origin. Ever since Mercator published his famous map in 1569, the conformal projection has proven to be very useful in accommodating earth's curvature while retaining the simplicity of using plane coordinates. The state plane coordinate systems in the United States are based on two variations of Mercator's projection – the Lambert conic conformal projection and the transverse Mercator projection. In each case, the angles are preserved, but the width of a state plane coordinate zone is limited to 158 miles to avoid distorting distances more than 1 part in 10,000. For the Lambert projection the 158-mile limitation is in the north-south direction while the 158-mile limitation on the transverse Mercator projection is in the east-west direction. Stated differently, the Lambert projection can extend long distances east-west without exceeding the distortion limit while the transverse Mercator projection extends a long distance north-south without exceeding the 1 in 10,000 distortion limit.

What is magic about the 1 in 10,000 limit? Nothing – the limit is arbitrary. Consider, when the state plane systems were designed in the early 1930's, routine local practice consisted of using a transit and steel tape for traversing. Generally such traverses had a ratio of precision of 1 in 5,000 to 1 in 10,000 and a systematic distance distortion up to 1 in 10,000 could be tolerated without detrimental consequence. That is no longer the case. Now better measurements can be made with equipment such as EDM, GPS, and photogrammetric mapping. Systematic error corrections for the grid scale factor are required to preserve the accuracy of modern measurements.

But, a flaw in the state plane coordinate systems (at least so far as surveyors are concerned) is that the elevation factor was not included as part of the design. The difference between a horizontal distance on the ground and the corresponding grid distance is given by the combined factor – the product of the grid scale factor and the elevation factor. True, in places where the elevation is within about 60 meters of sea level, the grid scale factor is the only difference that matters (at 60 meters the elevation factor is about 1 in 100,000). Otherwise, when working with grid and ground distances, both factors must be considered. Again, depending on the level of distortion that can be tolerated by the user, the grid/ground distance difference may not be an issue. But, in reality, the pendulum is swinging the other way because 1) modern equipment is used which can measure much better than 1 in 10,000 and 2) spatial data users are becoming more sophisticated. The grid/ground distance differences are important within smaller and smaller tolerances. Wouldn't it be nice to know that a meter is always a meter and not some distorted representation thereof?

Perhaps state transportation departments (DOT's) were the first to really address the grid/ground distance difference. What is required to make the coordinate inverse match the centerline stationing difference? For DOT's, centerline stationing represents horizontal distance and a grid inverse between the same two points which gives a different answer is of little use. In the past 50 years it has become common for state DOT's to design "project datum" coordinate systems in which the distance distortion is reduced 1) by multiplying the state plane coordinate values by an elevation ratio, 2) by raising the reference ellipsoid to the project elevation, or 3) by some other method. The result is that a grid coordinate inverse will match the centerline stationing difference within some (stated or unstated) tolerance. That is essentially the definition of a local coordinate system; the coordinate inverse is very nearly the same as (or identical to) the local tangent plane horizontal distance.

Contact your state DOT for information on specific projects or local policies. But, for a big picture overview, each reader is referred to Appendix III of "Using GPS Results in True 3-D Coordinate System" published in the February 1993, issue of the ASCE *Journal of Surveying Engineering*, Vol. 119, No. 1. In 1991 a questionnaire was sent to all 50 state DOT's asking how they handle the grid/ground distance difference. Replies from 46 out of 50 DOT's are tabulated in that appendix. An obvious conclusion after reading those replies is that there is little (or no) standardization of practice from one DOT to another. Can that lack of standardization be traced back to the state plane coordinate system design flaw? I believe it can. Two sources of distance distortion must be considered when designing a local coordinate system – the grid scale factor and the elevation factor. Both are mathematically well defined. The grid scale factor is related to the permissible width of the zone and the elevation factor is determined from the elevation of the terrain. Both should be considered when designing a local coordinate system.

At this point, two approaches should be considered. One is the traditional 2-D map projection solution; the other is a comprehensive 3-D solution. Both options are illustrated in Figure 6 of the February 1993, ASCE article. The 2-D approach is the "project datum" box shown in the lower left of Figure 6 and is discussed here. The other is the "P.O.B. datum" box, which will be the focus of a future article. The 3-D approach will show that the coordinate inverse distance is identical to the horizontal tangent plane distance, HD(1), as described in, "Computation of Horizontal/Level Distances," ASCE Journal of Surveying Engineering, Vol. 117, No. 3, August 1991.

The 2-D map projection approach is described in "Design of a Local Coordinate System for Surveying, Engineering, LIS/GIS," which can be found in the March 1993 ACSM Journal of *Surveying & Land Information Systems*, Vol. 53, No.1, pages 29-40. That paper contains a discussion of issues related to using state plane coordinates and the ground/grid distance difference. It also contains specific equations, which can be used to develop a tailor-made local projection using either a Lambert conic conformal projection or a transverse Mercator projection. Admittedly, the equations get a bit messy, but the procedures are explained, and examples are provided. Additionally, a DOS-based, menu-driven program is available gratis from the author, which can used to create a local projection and compute coordinate transformations. Regretfully, the software is designed for instructive purposes rather than for production efficiency. The program is a very powerful tool but is not what one would call user-friendly. To get a copy of the program, send an email to eburkhol@nmsu.edu and ask for "localcor." An execute file will be sent via return email.

The reader should also be aware that local coordinate systems can also be established using programs available from various software and equipment vendors. Just as there is no standardization between DOT's, one may find differences in local coordinates as computed by various software packages – even given the same design parameters. The suggestion in such a case is to obtain a copy of the algorithm used in the program and to find out exactly how the program works. Extensive testing may be required. A good place to start is to compare results of a local projection which uses the same parameters as a state plane system with the results one gets using NGS developed software such as CORPSCON. They should be identical if reference elevation is zero.

The "localcor" program has been tested extensively against the published algorithms and results of other programs such as CORPSCON (results are identical when using a reference elevation of 0.0 meters). The equations used in "localcor" are those listed in the ACSM paper. While great effort has been made to insure the veracity of the "localcor" program, it is not warranted in any way. Each user assumes responsibility for results obtained with the program.

Notes:

1. There is a typographical error in the equation for w_0 on page 37 of the March 1993 ACSM article. The last part of the equation should be

   $(U_4 + U_6 * \cos^2 f_0)$ and NOT $(U_4 + U_6 + \cos^2 f_0)$.

   The "plus" becomes a "times" and a matching closing parenthesis is needed.

2. The "localcor" program also contains provision for establishing local projections and computing transformations on an oblique Mercator projection. The algorithm for the oblique Mercator projection is not in the 1993 ACSM paper but is part of a paper, "State Plane Coordinates on the NAD 1983," presented by the author at the 1985 ASCE Convention in Denver, Colorado. That paper contains the algorithm, flowchart, and FORTRAN 77 source code for computing transformations on three projections; the Lambert conic conformal, the transverse Mercator (including UTM), and the oblique Mercator projection. A copy is available upon request from eburkhol@nmsu.edu.